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PROPAGATION OF SOUND IN LIQUIDS AND VISCOSITY

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ABSTRACT. An empirical relation $v=G\eta^{1.1}$ is given between sound velocity (v) and viscosity (η) of liquids. The propagation of sound in liquids is discussed on the basis of energy transference between molecules. The relation $v \propto \frac{1}{\sigma^9}$ between sound velocity and molecular diameter (σ) is also deduced theoretically therefrom. The relation $\eta \propto \frac{1}{\sigma^9}$ follows from above. Dispersion of acoustic velocity with frequency is shown to be true from the theory. The frequency range for dispersion is also discussed.

§ 1

It is well-known fact that the velocity of sound in a liquid varies with temperature and the viscosity of the liquid also varies rapidly with temperature. It is known that the velocity of sound depends upon the nature of movement of the molecules in a liquid and the viscosity of the liquid is dependent upon the nature of movement of the molecules under consideration. Both (sound velocity and viscosity) are known to be functions of the frequency of the molecules and their size. Then a relation may be expected between the two. The relation at first might seem too complicated but a closer investigation results in a simple relation.

It is seen that the ratio $\frac{\log v}{\log(\eta \times 10^5)}$ (where v is the velocity of sound in the liquid and η the viscosity of the liquid at that temperature) works out to be a constant independent of the liquid and independent of the temperature. This is shown by table I. The constant has a mean value of 1.2, where instead of using directly the value of η we use $\eta \times 10^5$, as η is a small quantity generally.

Therefore we have

$$\frac{\log_{10} v}{\log_{10}(\eta \times 10^5)} = 1.2. \qquad (1)$$

From this we get the relation

$$\log_{10} v = [\log_{10}(\eta \times 10^5)]1.2$$

or

$$v = (\eta \times 10^5)^{1.2}, \text{ independent of temperature.} \qquad (2)$$

TABLE 1

Liquid	Temperature °C.	Velocity Metres/sec.	Viscosity $\times 10^5$	$\frac{\log v}{\log(\eta \times 10^5)}$
Acetone	0	1273	395.0	1.212
	20	1190	300.0	1.240
	40	1100	280.0	1.242
Aniline	0	1712	805.0	1.104
	20	1650	440.0	1.218
	30	1606	349.0	1.282
	40	1570.5	241.0	1.343
	50	1540	189.0	1.401
Benzene	20	1324	649.0	1.111
	40	1231	492.0	1.149
Ether	0	1095	300.0	1.226
	10	1054	258.0	1.253
	20	1006	234.0	1.267
	30	949	212.0	1.277
Heptane	0	1235	519.0	1.139
	20	1154	410.0	1.171
	40	1070	344.0	1.196
Hexane	20	113(2, 3)	320.0	1.215
Chloroform	23	1000	550.0	1.096
Methylene iodide	24	977.7	475.0	1.124
Methyl alcohol	0	1187	813.0	1.056
	20	1121	591.0	1.117
	40	1056	450.0	1.139
Octane	0	1277	703.0	1.091
	20	1192	538.0	1.127
	40	1108	428.0	1.157
Toluene	0	1414	768.0	1.091
	20	1327.5	586.0	1.128
	40	1242	466.0	1.157
Carbon disulphide	23	1149	360.0	1.198

To make equivalence between dimensions of both sides of the relation there must be on one side of the equation a constant with the required dimensions. Thus

$$v = G_1[\eta \times 10^5]^{1\cdot 2} \qquad \ldots \quad (3)$$

where G_1 is a constant dependent on the nature of the liquid under consideration. Taking logarithms on both sides of (3) we get

$$\frac{\log_{10} v}{\log_{10}[\eta \times 10^5]} = \frac{\log_{10} G_1}{\log_{10}[\eta \times 10^5]} + \frac{11}{9}. \qquad \ldots \quad (4)$$

The above equation (4) will explain the small variation of the results in the last column of table I. In getting the results of table I, we have neglected and have not considered the effect of $\frac{\log_{10} G_1}{\log_{10}[\eta \times 10^5]}$. For ordinary liquids considered in Table I, the constant G_1 is very nearly equal to 1. Therefore the term containing G_1 will vanish and very nearly equals zero.

But for liquids having a high viscosity and relatively low velocity of sound the ratio $\frac{\log_{10} v}{\log_{10}[\eta \times 10^5]}$ will not be in line with those of the other liquids like those of table I. In such a case the constant G_1 will no more be nearly 1 but will be less than 1 and small. Therefore the term containing in (4) will be negative and the resulting value of the ratio will be small.

In the case of liquids of small viscosity and a relatively large velocity of sound the constant G_1 will be large and the term containing G_1 in (4) will have large positive value thereby increasing the value of the ratio $\frac{\log_{10} v}{\log_{10}(\eta \times 10^5)}$. For ordinary liquids the constant being very nearly 1, the effect of it is neglected and the value 1.22 or 11/9 is evaluated.

The following table II shows the case of highly viscous liquids like glycerine.

It will be seen from table II that for highly viscous substances the ratio attains a very low value.

Therefore we can write the equation in the final form as

$$v = G\eta^{\frac{11}{9}} \qquad (5)$$

where G is a constant.

Liquid	Temperature °C.	Velocity Metres/sec.	Viscosity	$\frac{\log v}{\log(\eta \times 10^5)}$
Glycerine	20	1923	8 5	.5526
Carbon tetrachloride	20	935	.0096	.9963
Butyl alcohol	23	1315	.0294	.8914

§2 PROPAGATION OF SOUND IN LIQUIDS

We can consider the propagation of sound in a liquid as due to a series of collisions between molecules initiated by the impinging wave. Let the sound wave in a liquid be divided into a number of parts each proportional to the intermolecular distance and let the total energy on the wave be divided into the same number of parts each proportional to the amplitude of the part ; then each part of the sound wave will have its associated energy proportional to its amplitude. If the wave reaches a molecule and the energy is absorbed by the molecule, the molecule will have an increased velocity to account for this increased energy. Then there will be an increase of the frequency of the molecule. Now assuming that at each vibration the molecule of higher energy gives its excess of energy to the neighbouring molecule, it will be seen that one part of the sound wave will travel the intermolecular distance (σ) in $1/\nu$ sec. If the sound wave be divided into x parts then λ parts or one wave will travel through $k.x\sigma$ in $1/n$ seconds, where n is the frequency of the wave, which is the same as v/ν seconds.

Now the velocity of sound is given by

$$v = n\lambda \qquad \ldots (6)$$

and

$$\frac{1}{n} = \frac{x}{\nu} \text{ or } n = \frac{\nu}{x} \qquad \ldots (7)$$

and from our assumption

$$k.x\sigma = \lambda \qquad \ldots (8)$$

where k is the proportionality constant dependent on the liquid. Then we get

$$v = k.x\sigma.\frac{\nu}{\lambda} \qquad \ldots (9)$$

$$v = k\nu\sigma. \qquad \ldots (10)$$

Assuming that

$$\nu \propto \frac{1}{\sigma^{m}} \qquad \ldots (11)$$

we get

$$v = k.\frac{1}{\sigma^{m-1}} \qquad \ldots (12)$$

Now putting $m = 10$

we get

$$v \propto \frac{1}{\sigma^{9}} \qquad \ldots (13)$$

Combining this with (5), we get $\eta \propto \frac{1}{\sigma^{7}}$ ... (14)

The *theoretical* relation (13) above is seen to be true empirically also [1] such that the assumptions made in deriving (13) are essentially correct.

In deriving (13) it is assumed that the frequency of a molecule is not changed very much. This changed frequency (which frequency must be used instead of the normal frequency in deriving (13)) will not vary much from the normal frequency of the molecules at that temperature. This change of frequency and its magnitude will be dependent upon the frequency of the sound wave. Therefore there will be dispersion of acoustic velocity at very high frequencies. The acoustic dispersion in the range we are considering will be very small and is neglected in deriving (13). The dispersion will be larger when the acoustic frequency is comparable with the molecular frequency. The theory *outlined* above is still in its earlier stages and has to be perfected to give quantitative results about dispersion and acoustic frequency.

From the above relations it is seen that much interesting facts can be deduced. It is possible to derive an equation for the temperature change of velocity of sound in liquids from (5). By advancing the theory proposed in §2 to solids the author hopes to get valuable information.

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